String-inspired ultraviolet/infrared mixing and preliminary evidence of a violation of the de Broglie relation for nonrelativistic neutrons

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We advocate a novel perspective on the phenomenology of a framework with spacetime noncommutativity which is of established relevance for string theory. Our analysis applies to cases in which the noncommutativity parameters are arranged according to the criteria of "light-like noncommutativity" and ultraviolet supersymmetry is assumed, so that the implications of the characteristic mechanism of ultraviolet/infrared mixing are relatively soft. We also observe that an analogous case of soft ultraviolet-infrared mixing is present in a previously-proposed Loop-quantum-gravity-inspired description of quantum spacetime. And we show that soft ultraviolet-infrared mixing produces an anomaly for the nonrelativistic de Broglie relation $\lambda v = h/m$, with correction term governed by a single (but particle-dependent) parameter χ. We test this hypothesis by comparing a determination of the fine structure constant that relies on the de Broglie relation for nonrelativistic neutrons to other independent determinations of the fine structure constant, and we obtain an estimate of χ that differs from 0 with four-standard-deviation significance.

In the long, and so far inconclusive, hunt for a quantum theory of gravity the main strategy was inspired by the discovery paradigm of the 20th century, the "microscope paradigm" with discovery potential measured in terms of the shortness of the distance scales probed. But it is actually expected that quantum gravity, besides certainly providing new phenomena in a far-UV (ultraviolet) regime, should also have significant implications in a dual IR (infrared) regime. For example, our present understanding of black-hole thermodynamics, and particularly the scaling $S \propto R^2$ of the entropy of a black hole of radius R, suggests that such IR features are inevitable [1, 2]. It is therefore perhaps no accident that noteworthy IR features were indeed found [3] in the study of the only theories in a "quantum spacetime" that proved to be manageable with presently-available methods of computation and interpretation, which is the case of theories with "canonical" noncommutativity of spacetime coordinates, $[x_\mu, x_\nu] = i\theta_{\mu\nu}$. Intriguingly it has also been robustly established that this type of spacetime noncommutativity emerges in various regimes [3] of String Theory, the most studied candidate for quantum gravity. For example, a mechanism analogous to the emergence of noncommutativity of position coordinates in the Landau model also characterizes the description of strings in presence of a constant Neveu-Schwarz two-form B-field [3].

We shall here focus on models with "light-like" noncommutativity matrix [4, 5] ($\theta_{\mu\nu}\theta^{\mu\nu} = \epsilon_{\mu\nu\rho\sigma}\theta^{\mu\nu}\theta^{\rho\sigma} = 0$) and assuming UV supersymmetry. The results of analyses of self-energy corrections [3, 6] in such light-like-$\theta_{\mu\nu}$ scenarios can motivate the study of modifications of the on-shell relation of the form

$$m^2 \simeq E^2 - p^2 + \chi_\theta \, m^2 \log\left(\frac{E + \vec{p}\cdot\hat{u}_\theta}{m}\right) . \qquad (1)$$

The parameter χ_θ takes different values for different particles [3, 6] and the unit vector $\hat{u}_\theta$ describes a preferential direction [4] determined by the matrix $\theta_{\mu\nu}$.

Our interest in the specific scenario (1) is in part motivated by the fact that it appears to characterize a type of IR behaviour which might also emerge in other approaches to quantum-gravity/quantum-spacetime research. This is suggested by the quantum-spacetime model of Refs. [7, 8], further developed[1] in parts of Refs.[9, 10], which was inspired by one of the competing perspectives on the semi-classical limit of Loop Quantum Gravity, and found corrections to the dispersion relation with dominant IR/long-wavelength behaviour linear in momentum [7, 8]. Clearly also (1) produces effects whose dominant IR behaviour is linear in momentum, since for small p ($E \simeq m$) one finds $\log[(E + \vec{p}\cdot\hat{u}_\theta)/m] \propto \vec{p}\cdot\hat{u}_\theta$. The long-wavelength behaviour of the two models therefore differs only because of the fact that invariance under spatial rotations (lost in (1)) is preserved by the scenario of Refs. [7–9]. In most of the following we shall consider simultaneously the two models, by observing that the characterization of (1) in terms of χ_θ and $\hat{u}_\theta$ is applicable, in the long-wavelength regime, to the scenario of Refs. [7–9] by replacing $\hat{u}_\theta$ with $\hat{p} \equiv \vec{p}/p$ (which correctly reproduces the space-rotation invariance assumed in Refs. [7–9]) and replacing χ_θ with an independent parameter $\chi_{\hat{p}}$, since any given set of data will constrain differently the strength of the effect in the two models (as a result of their different description of the fate of space-rotation symmetry).

While the merit of preserving space-rotation symmetry may appear to be a decisive argument in favour of the $\chi_{\hat{p}}, \hat{p}$ setup, we should stress that the case of Eq. (1) has the priority for what concerns the robustness of the

[1] In this first exploratory study we shall, like Refs.[9, 10], set aside the possibility of an helicity dependence [8] of the effects.

link to a candidate quantum-gravity theory, since its relevance for string theory has been investigated in detail [3, 4]. From that perspective it is perhaps useful for us to comment on alternatives to the choice of light-like $\theta_{\mu\nu}$ and UV supersymmetry. If the model completely lacks supersymmetry (even in the UV limit) then the IR features can have inverse-power-law dependence [6] on momentum, producing much more virulent IR effects than expected with (1). And if the $\theta_{\mu\nu}$ matrix is not of light-like type, one ends up either spoiling unitarity [3, 4] ("time-like noncommutativity") or producing IR features that exclusively involve the spatial components of momentum ("space-like noncommutativity"), in ways that produce a singular long-wavelength limit [3, 6].

In the very extensive (mostly string-inspired) literature devoted to spacetime noncommutativity, the presence of IR corrections, and the "UV/IR-mixing mechanism" [3, 6] that produces them, have attracted very strong interest. However, they were mainly contemplated as technically intriguing but physically cumbersome features. We shall here adopt a complementary perspective, perceiving their possible use as explicit examples of IR properties for the quantum-gravity realm as the key reason of interest in these models.

Consistently with the intuition we have been developing in an ongoing investigation of broader scopes [11], our characterization of the long-wavelength properties of theories in quantum spacetime focuses on modifications of the "nonrelativistic de Broglie relation" (whose standard form is $\lambda v = h/m$). We here derive such modifications assuming an on-shell relation of the form $m^2 \simeq E^2 - p^2 + \chi\, m\, \vec{p}\cdot\hat{u}$, which for $\chi = \chi_\theta$, $\hat{u} = \hat{u}_\theta$ describes the case of the long-wavelength limit of (1), while for $\chi = \chi_{\hat{p}}$, $\hat{u} = \hat{p}$ describes the case of the long-walength properties of the model of Refs. [7–9]. From such linear-in-momentum modifications of the on-shell relation one derives a modified nonrelativistic de Broglie relation which takes the form

$$\lambda\vec{v} \simeq \frac{h}{m}\hat{p} - \chi\frac{\lambda}{2}\hat{u} + O\left(\lambda v^3\right) \ , \qquad (2)$$

where we assumed that $v^j = \partial E/\partial p_j$ still holds.

By focusing on the nonrelativistic de Broglie relation the relevant phenomenology has the advantage, as we shall discuss in more detail elsewhere [11], of probing a rather robust feature of the type of models we are considering. In fact, modifications to $\lambda v = h/m$ are found both if $v(p)$ is anomalous ($v \neq p/(p^2+m^2)^{1/2}$) and if $v(p)$ is undeformed but there are deformations of the wavelength-momentum relation ($\lambda \neq h/p$, a modified "relativistic de Broglie relation"). Let us here illustrate this briefly within the example of the result of Refs. [7, 8], which was obtained by analyzing propagation of waves in the relevant quantum-spacetime picture, and therefore is most faithfully described in terms of a dispersion relation $4\pi^2 m^2/h^2 \simeq \omega^2 - k^2 + \chi_{\hat{p}}\, 2\pi\, m\, k/h$. From this one obtains Eq. (2) assuming undeformed wavelength-momentum relation ($k \equiv 2\pi/\lambda = 2\pi p/h$), and thereby obtaining from the group velocity, $\partial\omega/\partial k$, an anomalous dependence of velocity on momentum: $\vec{v} \simeq h\vec{k}/(2\pi m) - \chi_{\hat{p}}\,\hat{k}/2 \simeq \vec{p}/m - \chi_{\hat{p}}\,\hat{p}/2$. However, in the spirit of Ref. [12–14] and references therein, some authors assume that in such cases the relationship between velocity and momentum should still not be anomalous, advocating a consistency requirement between modifications of the $\omega(k)$ dispersion relation and modifications of the wavelength-momentum relation. In particular, for the model of Refs. [7, 8], following the scheme of Ref. [12], one could assume $\vec{k} \simeq 2\pi\vec{p}/h + 2\pi\chi_{\hat{p}} m\hat{p}/(2h)$, which produces and undeformed velocity-momentum relation: $\vec{v} \simeq h\vec{k}/(2\pi m) - \chi_{\hat{p}}\,\hat{k}/2 \simeq \vec{p}/m$. However, one then also finds $\lambda \equiv 2\pi/k \simeq h/(p + \chi m/2)$ which combines with $\vec{v} \simeq \vec{p}/m$ to produce once again (2).

An unpleasant aspect of the phenomenology we are proposing originates from the severe particle dependence of χ that one expects from the theory side, which limits our ability to identify a preferred type of particle probes. For the model based on spacetime noncommutativity, here parametrized with χ_θ, this particle dependence is derived rigorously [6] and is rather virulent: for a given quantum field the coefficient χ_θ depends on the strength and number [3, 6] of its interactions with other fields, including interactions with possible ultramassive fields (fields on which our current particle-physics laboratories can provide no information, but here relevant because of the UV/IR mixing [6]). Moreover, according to some perspectives on the quantum-gravity problem there might even be additional sources of particle dependence, linked to the compositeness of particles: various semi-heuristic arguments suggest [15, 16] that the short distance structure of spacetime should affect "composite particles" (clearly atoms, and perhaps even hadrons within a parton picture) in a way that gets softer for higher compositeness because of a sort of averaging-out/coarse-graining effect. We shall not dwell on these hypotheses here, but they did tentatively direct toward hadrons and leptons our search of examples of measurements that could be used to illustrate our proposal.

This literature search quickly informed us of the fact that, outside the quantum-gravity community, the de Broglie relation is by now an unquestioned cornerstone of the laws of physics. We found no reasonably recent review of tests of de Broglie relation. Even precision experiments assume it to be exactly valid, and often the analysis is reported in such a way that a reader can no longer disentangle the information on the role played by the de Broglie relation. Among the few exceptions to this (in our opinion unhealthy) state of affairs the most precise measurement we could find with an intelligible role played by the nonrelativistic de Broglie relation is the one of Ref. [17]. This concerns a study of neutrons which determined their wavelength λ, in terms of the d_{220} lat-

tice spacing [18] of high-perfection silicon crystals, and also their speed ($v \simeq 1600$ m/s). The final result can be given in the form [19]

$$\frac{\lambda v}{d_{220[W04]}} = 2\,060.267\,004(76)\ \mathrm{m/s}\ , \qquad (3)$$

where $d_{220[W04]}$ is the d_{220} lattice spacing of the silicon crystal WASO04 [18, 19], and numbers in parentheses are one-standard-deviation uncertainties in the last digits.

We observe that one can accurately test the nonrelativistic de Broglie relation for neutrons, $\lambda v = h/m_n$, by comparing the result (3) to experimental determinations of $h/(m_n d_{220[W04]})$ based on the formula [17]

$$\frac{h/m_n}{d_{220[W04]}} = \frac{1}{d_{220[W04]}}\ \alpha^2 \frac{m_e}{2R_\infty m_n}\ , \qquad (4)$$

where both m_e/m_n (ratio of electron and neutron mass) and the Rydberg constant R_∞ ($R_\infty \equiv \alpha^2 m_e/(2h)$) are very accurately known experimentally [17]. Through precision measurements of $d_{220[W04]}$ and of the fine structure constant α one can determine $h/(m_n d_{220[W04]})$ using (4) and check the agreement with the result (3), required for the validity of the de Broglie relation. This in turn would allow to deduce bounds[2] on χ, in the same spirit of other "quantum-gravity phenomenology" studies [20–22].

Of course, in Refs. [17, 19], and in papers that followed, the exact validity of the nonrelativistic de Broglie relation is assumed *a priori*, and the result (3) is mainly used for a determination of the fine structure constant (exploiting the relationship (4)), often denoted with α_{h/m_n}. The relatively recent Refs. [23–25] did compare α_{h/m_n} to other independent determinations of α, but their assessment of the situation was in part affected[3] by the related errata in Refs. [26, 27]. Both in (Fig.1 of) Ref. [23] and in (Fig.1 of) Ref. [25] the α_{h/m_n} result of Ref. [19] was described as fully consistent with the most accurate available determinations of α, but that comparison was implicitly affected by ignoring the erratum published in Ref. [26] which invalidates previous understandings of $d_{220[W04]}$. On the other hand, (page 1248 of) Ref. [24] pointed out a disagreement of 2.8-standard-deviation significance between α_{h/m_n} and independent measurements of α, but a key item for that analysis, concerning determinations of α based on measurements of the electron $g-2$, could not take into account the relevant erratum in Ref. [27].

For our assessment of the implications of the results for $\lambda v/d_{220[W04]}$ of Ref. [19] we rely on two recent results, which allow a very accurate determination of $h/(m_n d_{220[W04]})$. For α we rely on Ref. [28], which reported a remarkably accurate determination, $\alpha^{-1} = 137.035\,999\,084(51)$, obtained combining a measurement of the electron g–2 and an elaborate field-theory computation of the relationship between α and the electron $g-2$ within QED. The error in the QED computation that was reported in Ref. [27] has been resolved (the result is now fully confirmed by two independent derivations [28]) and the experimental uncertainty of the measurement of the electron $g-2$ has been reduced by a factor 3 with respect to the best previous measurements. For $d_{220[W04]}$ we rely on the very recent Ref. [29], which reported $d_{220[W04]} = 192\,015.570\,2(10)$ fm, a four-fold improvement in accuracy with respect to the best previous determinations.

Both of these improved measurements affect the analysis in the direction of a wider mismatch between $h/(m_n d_{220[W04]})$ and the result for $\lambda v/d_{220[W04]}$ obtained in Ref. [19]. As illustrated in Fig. 1, previous determinations of $h/(m_n d_{220[W04]})$ were already higher than $\lambda v/d_{220[W04]}$ of Ref. [19]. The recent precise determination of α reported in Ref. [28] is (moderately) higher than the value of α assumed in Ref. [23, 24], and this in turn produces a higher estimate of $h/(m_n d_{220[W04]})$. And the recent precise measurement of $d_{220[W04]}$ reported in Ref. [29] is noticeably smaller than the previous corresponding "world average", a reduction which also goes in the direction of further increasing the estimate of $h/(m_n d_{220[W04]})$. The net result is that our analysis exposes a four-standard-deviation discrepancy: combining the result (3) of Ref. [19], the determination of α recently reported in Ref. [28], and the measurement of $d_{220[W04]}$ recently reported in Ref. [29], we find $(h/m_n - \lambda v)/(h/m_n) = (1.46 \pm 0.37)\cdot 10^{-7}$. For the $\chi_{\hat{p}[n]}$ parameter ($\chi_{\hat{p}}$ for neutrons) this would imply

$$\chi_{\hat{p}[n]} = (1.55 \pm 0.39)\cdot 10^{-12}\ .$$

The lack of information on orientation of the apparatus for the data reported in Ref. [19] does not allow us to derive a definite estimate of $\chi_{\theta[n]}$. Still, unless one is willing to assume that by chance the data reported in Ref. [19] happened to sample in exactly uniform way all possible orientations of the neutron velocities (a "conspiracy' which, with a finite number of measurements, is even logically problematic), our findings would also imply, assuming the validity of Eq. (1), that $\chi_{\theta[n]} \neq 0$ with four-standard-deviation significance, also setting [11] a lower bound: $\chi_{\theta[n]} > 0.77\cdot 10^{-12}$.

While it is amusing to offer estimates of $\chi_{\hat{p}[n]}$ and $\chi_{\theta[n]}$ clearly what we exposed here is at best intriguing preliminary evidence of a violation of the nonrelativistic de Broglie relation, which would also admit description in models of such violations that are not of the type [11]

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[2] Note that for particles of speed $v \simeq 1600$ m/s one can rely on (2) (applicable for $v^3 \ll |\chi|$) only to probe $|\chi|$ not smaller than $\sim 10^{-15}$. To probe even smaller values of $|\chi|$ one should either include the leading relativistic correction or use slower particles.

[3] We gratefully acknowledge generous feedback by Francois Biraben and Francois Nez, which helped us becoming comfortable with our understanding of the limitations of the characterization of the comparison of α_{h/m_n} and α found in Refs. [23–25].

we here focused on. And even the apparent crisis for the nonrelativistic de Broglie relation should of course be perceived with healthy skepticism, particularly considering the magnitude of the implications for our present description of the laws of physics. Still we do feel that, indeed because of the paramount importance that such a discovery would carry, this small crisis should be promptly investigated experimentally.

It seems that priority should be given to investigations of the most vulnerable link in our characterization of the discrepancy, which is the fact that the original result of Refs. [17, 19] has not been checked in a repetition of the experiment. The final estimate reported in Ref. [19] conscientiously combined (throughout-consistent) data gathered over two stages of measurements, separated by a full reinstallation and upgrade of the apparatus, and for five somewhat different configurations. But with some of the novel techniques developed over this past decade (see, *e.g.*, Ref. [29] and references therein) a significant improvement of the result should be easily within reach. In light of the particle dependence of the effect expected from the theory side, and of the prudence that from a more general perspective our present (lack of) understanding of the quantum-gravity problem demands, it would be ideal to have new precision tests of the de Broglie relation again for neutrons, and for velocities of the neutrons comparable to the ones produced in the setup of Refs. [17, 19]. It is certainly also interesting to search for analogous effects for atoms, especially since in some cases (such as caesium, $\chi_{[Cs]}$, and rubidium, $\chi_{[Rb]}$) very small values of χ could perhaps be probed. A sizable difference between $\chi_{[n]}$ and χ of atoms with large mass number might not be surprising, but according to our personal theoretical prejudice relatively small differences should be found between the case of neutrons and the case of Hydrogen atoms.

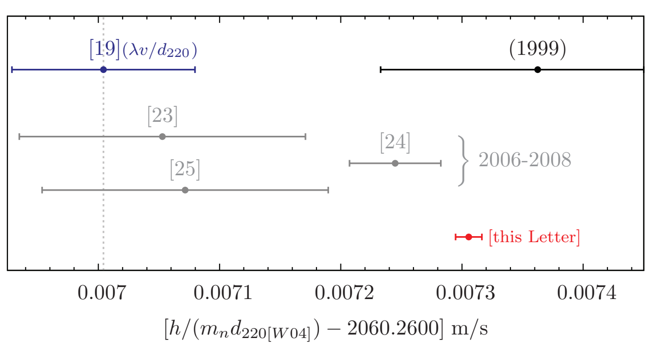

FIG. 1: We compare the result for $\lambda v/d_{220[W04]}$ of Ref. [19] to some estimates of $h/(m_n d_{220[W04]})$ that (explicitly or implicitly) were provided in the literature. There was reasonably good agreement in 1999 (publication of [19]). From Refs. [23], [24] and [25], published between 2006 and 2008, one could obtain different estimates of $h/(m_n d_{220[W04]})$, all affected in part by the errata in [26] and/or [27]. Our updated determination of $h/(m_n d_{220[W04]})$ simply combines recent precise measurements of $d_{220[W04]}$ (Ref.[29]) and α (Ref.[28])

Even if, as it is natural to expect, the experimental situation eventually settles removing the anomaly here exposed, our study could still inspire at least a small shift of focus for quantum-gravity theory research: as here illustrated, there might be windows of opportunity for quantum-spacetime phenomenology in precision measurements of properties of the long-wavelength regime, and perhaps a bit more of the theory effort could be diverted from the "microscope paradigm" toward targeting these possible opportunities.

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